Correlational and Non-Extensive Nature of Carbon Dioxide Pricing Market

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Abstract

In this paper, at the first time, the analysis of correlational and non-extensive properties of the CO_2 emission market relying on the carbon emissions futures time series for the period 04.07.2008-10.05.2021 is performed, and the daily data of the power sector from the U.S. Carbon Monitor for the period 01.01.2019-10.05.2021, which consist the data of both individual countries (USA, Germany, China, India, United Kingdom, et al.) and global emissions (World) are investigated using such approach. To demonstrate the applicability of these methods on systems of another nature and complexity, the analysis of the Dow Jones Industrial Average (DJIA) index is presented. The results show that both futures and the DJIA are presented to be non-extensive, and the distribution of their normalized returns can be better described by power-law probability distributions, particularly, by q-Gaussian. Tsallis triplet for the entire time series of CO_2 emissions futures and the DJIA is estimated, and q-triplet as an indicator of crisis phenomena is presented, relying on the sliding window algorithm. It can be seen that the triplet behaves characteristically during economic crises. This study shows that the toolkit of the random matrix theory (RMT) allows to investigate the correlational nature of the carbon emissions market and to build appropriate indicators of crisis phenomena, which clearly reflect the collective dynamics of the entire research base during events of this kind.

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1. Introduction

Global climate change and carbon pricing are increasingly popular topics. The ecological security of the earth and the long-term forecasting and tracking of the carbon markets development have become a new challenge facing all countries that are interested in environmental development. It also should be noted that climate change is such a challenge that must be considered not only by the local groups but with coordination at the international level. According to the 2015 Paris Climate Agreement [1], nowadays *low-carbon economy* has been supported through regulation of emission trading schemes, taxes, and fossil fuel extraction fees.

Carbon pricing is such an indicator for countries and companies that forces them to switch to more efficient processes or cleaner fuels. For governments, it is a guiding mechanism for dealing with carbon dioxide emissions. More support and awareness around carbon pricing leads to significant costs for companies, amounting to as much as $1.3 trillion from the 2030 year across companies in the S&P 500.

On the other hand, it forces market mechanisms to produce financial incentives to lower emissions by switching to more energy-saving and emission reduction technologies [2]. In the emerging class of energy and carbon hedge funds, policymakers, risk managers, and emission intensive firms need to track the efficiency of the carbon market [3].

Carbon markets are presented to be similar to other financial markets, such as securities and foreign exchange markets. At the same time, such carbon market as China‘s, except common influence factors of the traditional markets, is influenced by fossil energy price, quota allocation system, and extreme weather change [4].

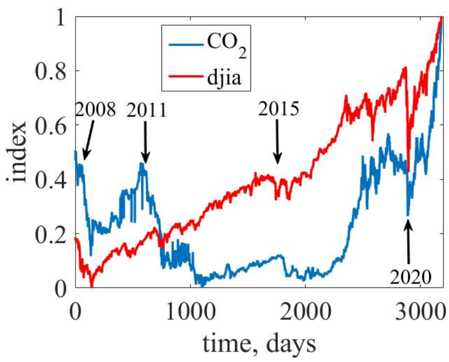
Thus, it seems that the carbon market is a complex and self-organized system, consisting of a plurality of interacting agents possessing the ability to generate new qualities at the level of macroscopic collective behavior. Its dynamics can be tracked and forecasted, in most cases, from the complex network of market agents or as an integrated output signal – a time series of carbon prices.

For carbon pricing, it is important to have the risk identification and forecasting system to have the opportunity for implementation responsive laws and innovative approaches in advance for sustainable economical development. Prigogine‘s manifestations of the system complexity [5] is an idea by which we will be guided during studying the carbon market and appropriate *quantitative measures of complexity*. The key idea here is the hypothesis that the complexity of the system before the crashes and the actual periods of crashes must change. This should signal the corresponding degree of complexity if they are able to quantify certain patterns of a complex system.

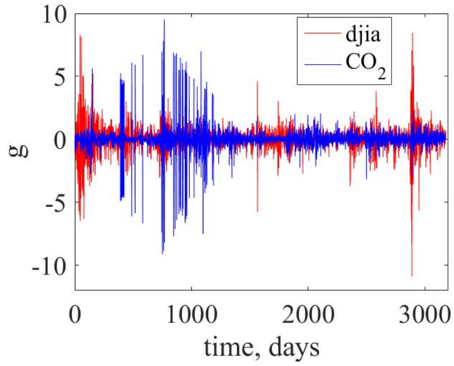
Previously, some of such quantitative measures of complexity for cryptocurrencies, stock, and sustainability indices [6, 7, 8, 9, 10, 11, 12] were studied. In this paper, in order to have the possibility to study trading opportunities, the prospects for investing in a market, particularly, to study various components that define the nature of carbon prices and the collective behavior of the whole carbon market which is of particular value for politicians of specific countries, such informative measures of complexity as *Tsallis statistics* and *Random matrix theory* are presented.

Further, the daily prices of carbon regarding carbon emissions futures time series (Investing.com) for the period 04.07.2008-10.05.2021 are analyzed. Moreover, the dynamics of emissions is depended on the economical situation of a specific country and the whole world. In order to present the validity of the presented methods, for comparison, the Dow Jones Industrial Average (DJIA) index is selected as the most quoted financial barometer that has become synonymous with the financial market in general. The DJIA data were obtained for the same period (Yahoo! Finance).

Fig. 1a clearly shows the correlations of the time series, especially in periods of crisis.



(a)



(b)

Figure 1: Comparative dynamics of CO_2 and the DJIA daily values (a). Standardized returns of CO_2 and the DJIA (b).

However, in Fig. 1b, these correlations are no longer obvious for standardized returns. Such a feature of the behavior of standardized returns causes the specific dynamics of the complexity measures under consideration, which are calculated based on the returns data or their modules. CO_2 returns in turn are calculated as $G(t) = \ln x(t+\Delta t) - \ln x(t) \cong [x(t+\Delta t) - x(t)]/x(t)$, and standardized returns can be defined as $g(t) \cong [G(t) - \langle G \rangle]/\sigma$, where $\sigma \equiv \sqrt{\langle G^2 \rangle - \langle G \rangle^2}$ – standard deviation of G, Δt is a time lag (in our case $\Delta t = 1$), and $\langle \ldots \rangle$ – the mean value of the time period under study.

As a database for the study of correlation processes in the carbon dioxide emission markets, we used the daily data of the power sector from the U.S. Carbon Monitor for the period 01.01.2019-10.05.2021. It contains data of the most active emitters of carbon dioxide, including both individual countries (USA, Germany, China, India, United Kingdom, etc.) and global emissions (World).

Finally, to study the dynamics of emissions and stock indices and indicators, the sliding window procedure is used. All the procedures below will be carried out within a subset of the length w_{win}, after which the window is shifted by the predefined time step h_{win}, and the corresponding algorithms are repeated until the entire series is completely exhausted.

All indicators were calculated using author's software and libraries based on Matlab programming language [13].

Therefore, our paper is structured as follows. Section 2 emphasizes studies that have been dedicated to this market and different methods of complexity applied to it. Section 3 describes the main instrument for studying non-extensive nature of the carbon market. Section 4 provides the idea of Random matrix theory. Section 5 is the conclusion of this paper.

2. Literature review

In recent years global warming and its influence factors have attracted widespread attention. In the paper [14], authors emphasize that numerous number of papers were devoted to household CO_2 emissions. Such research is presented to be interdisciplinary and, according to the author‘s study, have to consider overall cognition of the environment, the economy, society, and technology.

As an example of interdisciplinary methods, Li et al. [15] study the global carbon transfer evolution in terms of complex networks. They use the MRIOA model to measure the heterogeneous carbon flow connections, detect regional cluster structures and identify each economy's coreness value in the context of the core-periphery model. Their empirical results can give new insights on global carbon flow patterns, give reliable footprint indicators and consumption-based models for policymakers. Jiang et al. [16] imply complex network, panel regression, and multi-regional input-output analyses to determine the influence of different countries in the global carbon emissions embodied in trade transfer networks on their direct carbon emissions. Results present that countries' role in the embodied carbon emission transfers changed over time. Such an approach gives the possibility to look at the factors by which direct carbon emissions are ruled and the dependence of direct emissions from embodied.

Wang et al. [17] investigate cross-correlation between energy and emission markets from the perspective of multifractal analysis. Using the detrended cross-correlation analysis and its multifractal extension, they examine power-law cross-correlations and find three returns of oil, gas, and CO_2 are fat-tailed and obey "inverse cubic power-law". Generally, the nonlinear and multifractal behavior are peculiarities of individual and cross-correlated emission and energy markets. Applying the sliding window approach, they show dynamically how changes their multifractal nature during different periods. Zou and Zhang [4] have done a similar study on the time series of domestic energy and carbon markets in China. They have found that these markets are correlated and present multifractal characteristics: long-term memory and fat-tailed probability distribution of their returns. Depending on economical and political situations, their relationship shows different trends, multifractal characteristics, and correlations change. Analyzing high-frequency time series of air measurements, Karatasou and Santamouris [18] applied power spectral density analysis over time scales and found that air temperature data exhibit turbulent-like intermittent properties with multifractal statistics. Multifractal nature has not spared the soil CO_2 emissions and selected soil attributes: soil water content, temperature, clay content, macro and microporosity, air-free porosity, magnetic susceptibility, bulk density, humification index of soil organic matter, and relative to organic carbon content [19]. EU carbon market also demonstrates multifractality [20].

Techniques from recurrence analysis [21, 22] are another solution to how to interact with the complexity of the system and, particularly, with recurrence dynamics. Kisel'ák et al. [23] analyze methane and carbon dioxide emissions using recurrence plots. Both CO_2 and CH_4 presents deterministic, stochastic, and chaotic periods from recurrence plots and quantitative measures of recurrence quantification analysis. Sparavigna [24] presents recurrence plots to study the dynamics of CO_2 concentration and emissions in metric tons per capita of US, China, Italy, UK, Japan, and Canada. Different interesting patterns in recurrence plots were explored and found similarities in trends of several countries.

Information entropy and its extensions is another solution for accessing dynamics of CO_2 emissions. Suh [25], using cross-entropy, reveal inequality in the regional distributions of carbon dioxide emissions in the U.S, namely, between-region and within-region inequalities. Their entropy-based model demonstrates that these inequalities vary across the regions. Alptekin et al. [26] refers to 28 EU countries and Turkey and nine low carbon development indicators. Using information entropy method, authors weights of importance in grey relational analysis. Also, they find how the importance of each indicator changes for different years.

In the following sections, non-extensive statistics and random matrix theory will be presented.

3. Non-extensive statistics and Tsallis triplet

Non-extensive statistical theory mathematically basing on non-linear equation

$$\frac{dy}{dx} = y^q, \; (y(0) = 1, q \in \Re)$$

and generalized definition of entropy

$$S_q = -k\frac{1 - \sum_i p_i^q}{1-q},$$

which is defined, regarding the q-exponential function

$$e_q(x) = \begin{cases} (1 + (1-q)x)^{\frac{1}{1-q}}, & \text{if } 1 + (1-q)x > 0 \\ 0, & \text{if } 1 + (1-q)x < 0 \end{cases}$$

and q-logarithm

$$\ln_q(x) = \frac{x^{1-q} - 1}{1-q}.$$

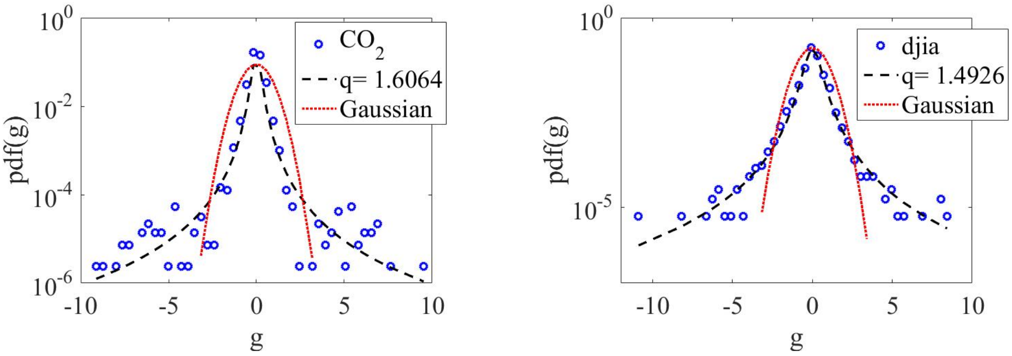
While the Tsallis entropy S_q measures the complexity, the power-law exponent q characterizes the degree of correlations (non-extensivity) of the system.

Considering two probabilistically independent systems A and B, their property of non-additivity can be expressed as

$$S_q(A + B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B).$$

The first part of the equation is additive while the second is multiplicative, describing the long-range interactions between the two systems.

For $q \to 1$, non-extensive statistics reduces to usual Boltzmann-Gibbs (BG) statistics (equilibrium state), which consider systems with short-range correlations inside their immediate neighborhood and close to a Gaussian state. But the real-world (non-extensive) systems such as stock or emissions futures are presented to be far from a simple Gaussian state. Figs. 2a and 2b demonstrate that studied signals are far away from additivity, their q-exponents > 1, and they are characterized by fat-tails that can be better described in terms of non-extensive statistics, particularly, by q-Gaussian distribution.



(a) (b)

Figure 2: Comparison of empirical distributions for CO_2 (a) and DJIA (b) time series with Gaussian and q-Gaussian distribution. In addition, the parameter of non-extensivity is defined for both cases.

For a non-extensive system, the value of the index depends on the estimated properties of the dynamics and phase space of the system. For dynamical systems that follow non-extensive statistics, a q-triplet is evaluated. These indices can describe such features as q-exponential sensitivity to initial conditions (weak chaos, described by growth with a parameter q_{sens}), q-exponential relaxation of macroscopic quantities towards equilibrium (exponential decay with a relaxation parameter q_{rel}), and q-exponential distribution describing a metastable or quasi-stationary state which can be described with a parameter q_{stat}. $(q_{stat}, q_{sens}, q_{rel}) \neq (1, 1, 1)$ have to satisfy the condition $q_{sens} \leq 1 \leq q_{stat} \leq q_{rel}$.

Table below presents the values of q_{stat}, q_{rel}, and q_{sens} for the entire data of dioxide futures and the DJIA index. We can see that both time series have to be described in terms of non-extensive statistics.

Table 1
Characteristic exponents for the entire time series of CO_2 and the DJIA.

Index	q_{stat}	q_{rel}	q_{sens}
CO_2	2.25	1.88	0.23
DJIA	1.60	2.40	-0.29

But, since the complexity of the system will naturally change, as was mentioned in the introduction, further calculations will be performed within the framework of the sliding window.

The calculations were carried out taking into account various window lengths w_{win} and time steps h_{win}. If h_{win} is small, there are too few values for constructing the indicators. In the other case, if the window is too big, then the differentiation of all crises seems problematic. In a large range, we can cover several crises simultaneously, the dynamics of which will affect the accuracy of the indicators. Also, with a large time step, it is possible to skip the period, which may seem to be the cornerstone in identifying a further crisis. According to the results of modeling, $w_{win} = 250$ and $h_{win} = 1$ seems to be a reasonable choice.

3.1. Tsallis q-stationary parameter

The value of q_{stat} for the stationary state is derived from probability distribution function (PDF) of returns, which in turn is obtained by fitting q-Gaussian

$$P_q(\beta; r) = \frac{\beta}{C_q} e_q(-\beta r^2),$$

where β is a positive number and C_q is a normalization constant, and C_q has the form:

$$C_q = \frac{\sqrt{\pi}\,\Gamma(\frac{3-q}{2(q-1)})}{\sqrt{q-1}\,\Gamma(\frac{1}{1-q})} \quad \text{for} \quad 1 < q < 3.$$

By minimizing the $\sum_i [P_q(\beta; r_i) - p(r_i)]^2$, we select β.

Further calculations use a time series of absolute returns obtained from standardized one, as well as returns change $\Delta G(t) = |G(t + \Delta t)| - |G(t)|$.

The first value from the triplet, the stationarity index q_{stat}, is calculated based on the probability distribution of returns change $\Delta G(t)$. To obtain the distribution, the interval $[\min(\Delta G(t)), \max(\Delta G(t))]$ is divided into subintervals $h_{\Delta G} = [\max(\Delta G) - \min(\Delta G)]/N_{\Delta G}$, where $N_{\Delta G}$ is the specified number of expected intervals.

Next, the number of $\Delta G(t)$ values that fall into each subinterval $(r_i, r_{i+\Delta t}) = (r_i, r_i + h_{\Delta G})$ is counted, which is then divided by the total number of returns change. As a result, a set of paired values $r_{i+1/2}, p(r_{i+1/2})$ is formed, where $p(r_{i+1/2})$ is an element of the probability distribution obtained for $r_{i+1/2}$ that is the middle of the corresponding interval.

Then, the index q_{stat} is found from the best linear adjustment in a $\ln_q[p(r_{i+1/2})]$ vs. $r^2_{i+1/2}$ graph, varying the index q between 0.5 and 7.0 with the step h_{stat} (preferably, $h_{stat} = 0.01$ or, to speed up the calculation procedure, $h_{stat} = 0.1$).

Fig. 3 demonstrates comparative q_{stat} dynamics for carbon emissions and the DJIA.

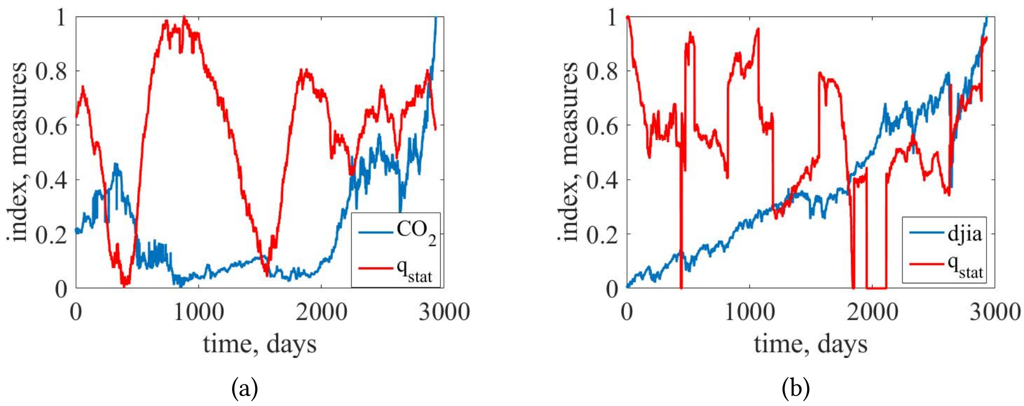


Figure 3: Comparative dynamics of q_{stat} with carbon emissions futures prices (a) and the DJIA (b).

3.2. Tsallis q-relaxation parameter

The corresponding q-value for the relaxation process is obtained from the autocorrelation coefficient

$$A(\tau) = \frac{\sum_t |g_{t+\tau}| \cdot |g_t|}{\sum_t |g_t|^2}.$$

For BG statistics, this correlation should decrease exponentially. However, the autocorrelation of financial time series for absolute values of returns (volatility) always decreases much more slowly. In addition, as the time series is presented to be non-stationary, the correlation coefficient should change over time. Thus, the corresponding q_{rel} index should also change over time. Similarly to previous approach, the value of q_{rel} can be estimated by best fit on $\ln_q A(\tau)$ vs. scale τ.

Fig. 4 demonstrates comparative q_{rel} dynamics for carbon and the DJIA.

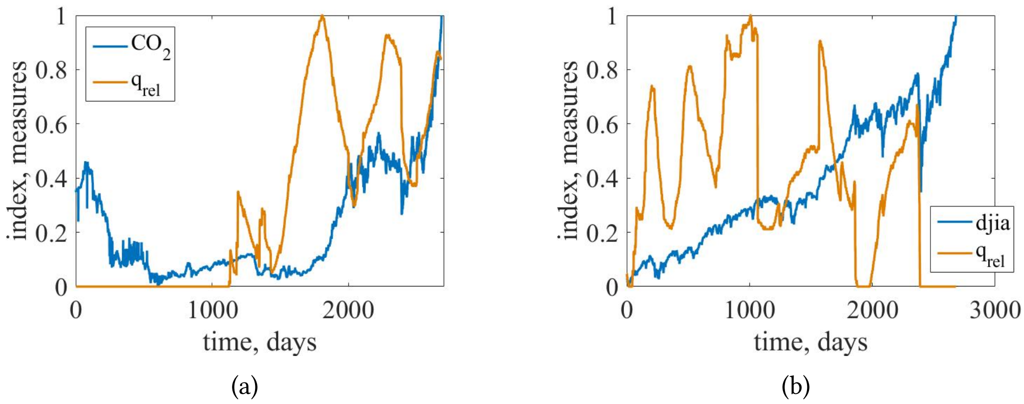


Figure 4: Comparative dynamics of q_{rel} with carbon emissions futures prices (a) and the DJIA (b).

3.3. Tsallis q-sensitivity to initial conditions

Systems with weak chaos (power law sensitivity to initial conditions) are better described by the non-extensive statistics. Deviations of the neighboring trajectories of the attractor lead to multifractal structure of the studied system [27]. Initially, it was hypothesized, and later proved for time series of non-intensive systems of different nature, that a relation occurs [28]:

$$\frac{1}{1 - q_{sens}} = \frac{1}{\alpha_{min}} - \frac{1}{\alpha_{max}},$$

where α_{min} and α_{max} are the extreme values of the multifractal spectrum $f(\alpha)$ for which $f(\alpha) = 0$, and α is the local scaling exponent (the singularity strength or Hölder exponent) [29]. $f(\alpha)$ denotes the fractal dimension of the local attractor's subset and can be calculated using the Multifractal Detrended Fluctuation Analysis (MF-DFA) method [30].

Fig. 5 illustrates comparative q_{sens} dynamics for carbon and the DJIA.

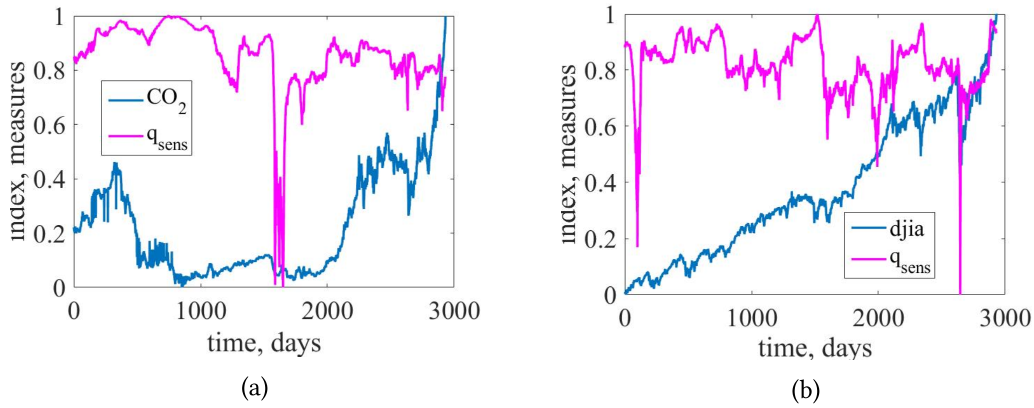


Figure 5: Comparative dynamics of q_{sens} with carbon emissions futures prices (a) and the DJIA (b).

4. Random matrix theory

Determining correlations between different stocks is a topic that is interesting not only from the scientific reasons for understanding the economy as a complex dynamic system but also from a practical point of view, in particular, from the point of view of asset allocation and portfolio risk assessment. We will analyze mutual correlations between stocks using the concepts and methods of random matrix theory used in the context of complex quantum systems, where the exact nature of interactions between subunits is unknown.

RMT [31, 32, 33] is a popular technical tool for investigating the cross-correlation in financial [34, 35, 36, 37] and energy markets [38]. The random matrix theory mainly studies some statistical properties of the eigenvalues and eigenvectors of the random matrix.

To quantify correlations, first of all, we define standardized returns of the i^{th} emissions at time t. Then the calculation of the cross-correlation matrix C is reduced to the calculation of the formula $C_{ij} \equiv \langle g_i(t) g_j(t) \rangle$.

By construction, the elements C_{ij} are restricted in the domain $-1 \leq C_{ij} \leq 1$, where $C_{ij} = 1$ corresponds to perfect correlations, $C_{ij} = -1$ corresponds to perfect anti-correlations, and $C_{ij} = 0$ corresponds to uncorrelated pairs of energy prices.

Difficulties appear as the analyzed pair of energy commodities is presented to be non-stationary, and the shorter the length, the less accurate mutual correlations between series. It is thus important to devise methods that allow one to distinguish "signal" from "noise", i.e. eigenvectors and eigenvalues of the correlation matrix containing real information (which one would like to include for risk control), from those which are devoid of any useful information, and, as such, unstable in time. From this point of view, it is interesting to compare the properties of an empirical correlation matrix C to a "null hypothesis" purely random matrix as one could obtain from a finite time series of strictly independent assets. If the properties of C correspond to the properties of a random matrix, then we can say that the empirically measured correlations are random. In contrast, the deviations from the random matrix case might suggest the presence of true correlations ("information").

4.1. The distribution of eigenvalues

For getting the mutual information between assets, formula for the cross-correlation matrix can be symbolically rewritten as $C = \frac{1}{L} G G^{\intercal}$, where G is the matrix of size $N \times L$ with elements $\{g_{im} = g_i(m\Delta t);\ i = 1, \ldots, N;\ m = 0, \ldots, L-1\}$ and $^{\intercal}$ denotes matrix transportation. Let's consider random (shuffled) correlation matrix $R = \frac{1}{L} A A^{\intercal}$, where A is $N \times L$ rectangular matrix that consists N time series with L random values a_{im}, mean 0 and variance $\sigma^2 = 1$.

For standardized logarithmic returns of the i^{th} emissions, pairwise cross-correlation coefficients between any two returns time series are calculated. Further estimations will also consider sliding window procedure, where corresponding $w_{win} = 50$ and $h_{win} = 1$. Graphical representation of correlation coefficients between CO_2 emissions of different countries is presented in Fig. 6.

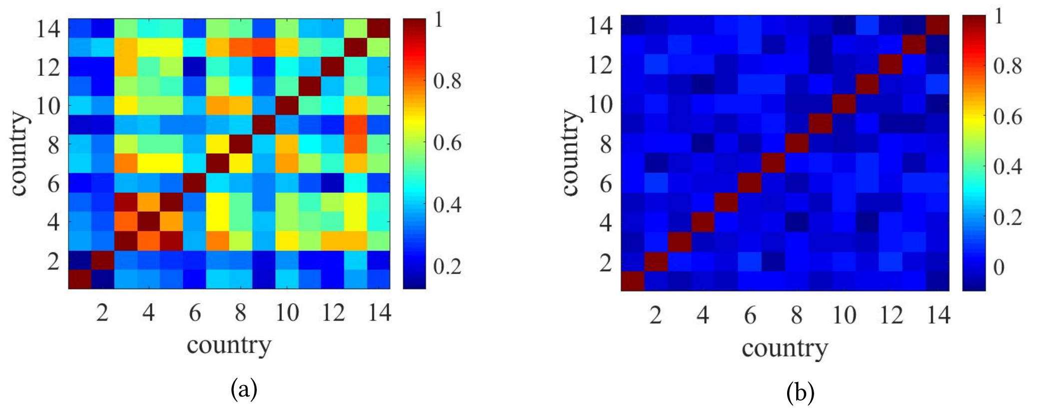


Figure 6: Heatmaps of the initial (a) and shuffled (b) correlation matrices.

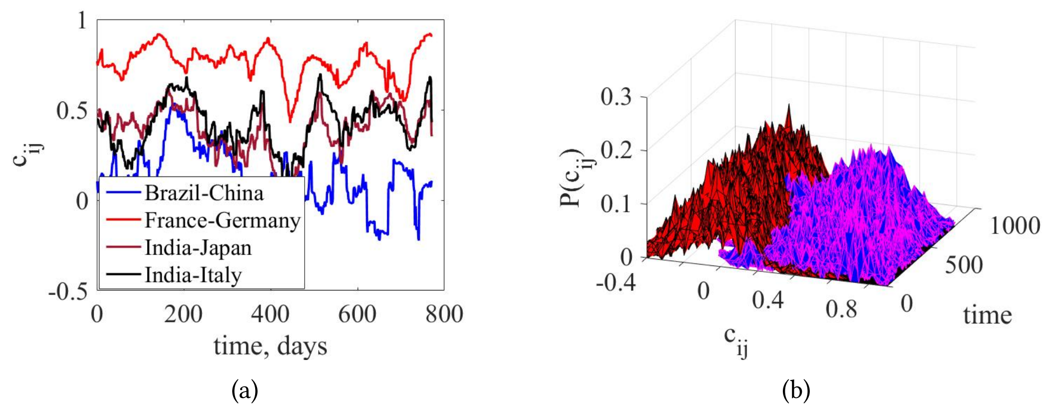


Figure 7: The dynamics of pairwise correlation coefficient of countries included to database with the sliding window technique (a). The windowed distributions of pairwise correlation coefficients for initial (purple) and shuffled (red) matrices (b).

From Fig. 7b it can be seen that the distribution of the paired correlation coefficients of the initial database differs significantly from the distribution function described by the RMT. It is noticeable that CO_2 emissions of different countries appear to be significantly correlated and self-organized systems.

The statistical properties of random matrix R are known. Particularly, as $N, L \to \infty$, such that $Q \equiv \frac{L}{N}(\geq 1)$ is fixed, the probability density function P_{rm} of eigenvalues of the random correlation matrix is given by

$$P_{rm}(\lambda) = \frac{Q}{2\pi\sigma^2} \frac{\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{min})}}{\lambda}$$

with $\lambda \in [\lambda_{min}, \lambda_{max}]$, where λ_{min} and λ_{max} are the largest and the smallest eigenvalues of R and, correspondingly, $\lambda_{min}^{max} = \sigma^2(1 + 1/Q \pm 2\sqrt{1/Q})$.

In further, the distribution of eigenvalues $P(\lambda) = \frac{1}{N}\frac{dn(\lambda)}{d\lambda}$ of C with $P_{rm}(\lambda)$ will be compared. Now, in Fig. 8 the spectrum of eigenvalues and the averaged correlation coefficient in the framework of the moving window approach is presented.

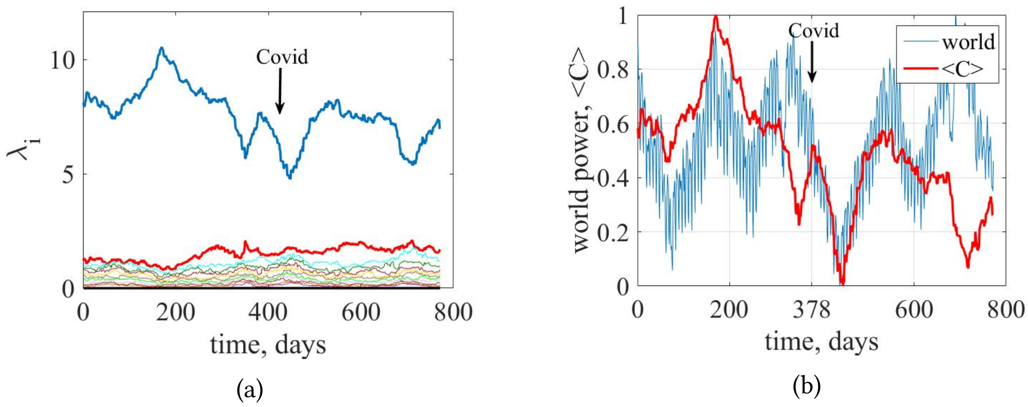


Figure 8: Window dynamics of the eigenvalue spectrum of the initial correlation matrix (a). The exact time series of global emissions and the averaged correlation coefficient (b).

A comparison of the dynamics of the λ_{max} (Fig. 8a) and the correlation coefficient (Fig. 8b) shows their practical identity. The arrow "Covid" indicates the beginning of the crisis associated with the coronavirus pandemic. The crisis leads to a decrease in CO_2 emissions and, accordingly, the degree of correlation of the market under study. Fig. 8a shows that the maximum eigenvalue is the most informative indicator, and it also changes over time.

Modes of the market can be reflected in eigenvalue and eigenvector pairs of the empirical correlation matrix C. Eigenvectors correspond to the *participation ratio* (PR) and its *inverse participation ratio* (IPR)

$$I^k = \sum_{l=1}^{N} [u_l^k]^4,$$

where $u_l^k, l = 1, \ldots, N$ are the components of the eigenvector u^k. So PR indicates the number of eigenvector components that contribute significantly to that eigenvector. More specifically, a low IPR indicates that all assets move in a similar fashion, responding to the overall trend of the market. In contrast, a large IPR would imply that the factor is driven by the dynamics of a small number of assets. The irregularity of the influence of the eigenvalues of the correlation matrix is determined by the *absorption ratio* (AR)

$$AR_n = \frac{\sum_{k=1}^{n} \lambda_k}{\sum_{k=1}^{N} \lambda_k},$$

which is a cumulative risk measure which measures the fraction of the overall variance in returns explained (absorbed) by a subset of eigenvalues.

Figs. 9a and 9b present how differ $P(\lambda)$ and IPR from predictions of RMT.

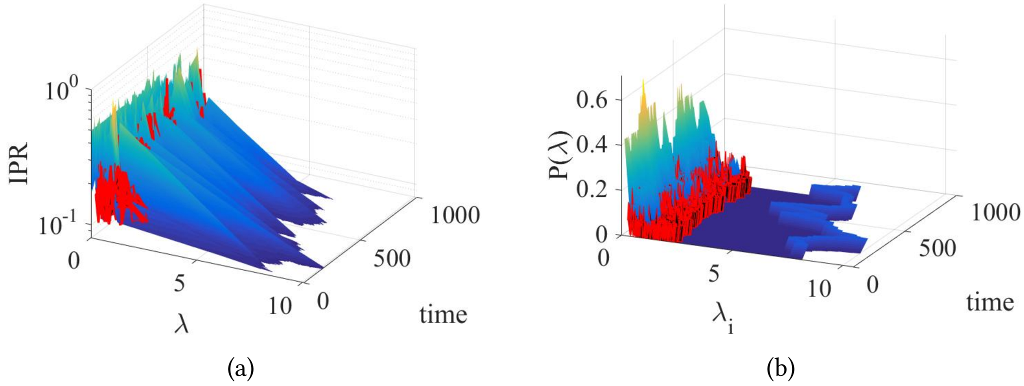


Figure 9: The distribution of IPR (a). The eigenvalue spectrum distribution function (b). The results obtained for the random matrix are highlighted in red.

Next, IPR, λ_{max}, and AR are calculated using sliding window approach and presented in Fig. 10.

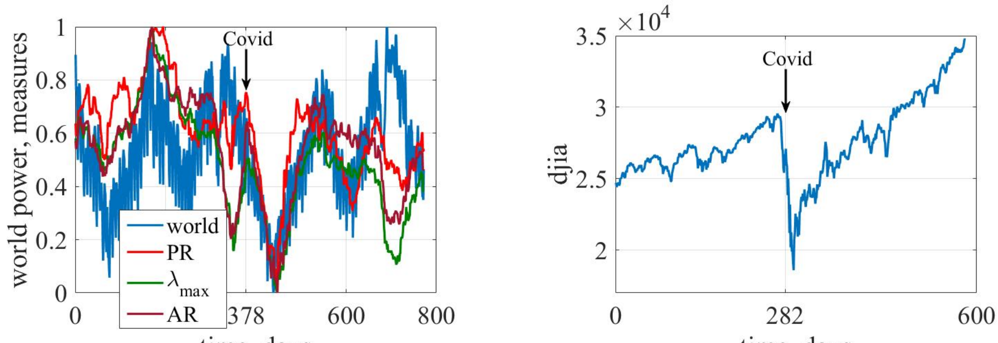


Figure 10: The dynamics of global CO_2 emissions along with PR, λ_{max}, and AR.

5. Conclusions

In this paper, the correlational and non-extensive properties of the CO_2 emission market are analyzed for the first time on the example of futures and the U.S. Global Carbon Monitor data. It is shown that the distribution of normalized returns for the dioxide futures obeys the Tsallis statistics ($q > 1$). As an example, we took the DJIA index to test the applicability of the presented methods for which $q = 1.4926$. Tsallis triplet is also calculated for CO_2 and DJIA. Obviously, the changing dynamics of the initial time series should lead to varying values of the triplet. It is shown that the dynamics of the values of q-triplet change in a characteristic way during economic crises. Although this dynamics is different for carbon emissions and the DJIA, it captures the changing trends in both markets. As expected, the carbon market is strongly correlated and its properties calculated by the RMT method allowed us to identify a number of characteristic measures (λ_{max}, absorption ratio, etc.), which are indicators of crisis phenomena in this market.

A significant advantage of the introduced measures is their dynamism, i.e., the ability to monitor the change in time of the chosen measure and compare it with the corresponding dynamics of the output time series. This allowed us to compare the critical changes in the dynamics of the system, which is described by the time series, with the characteristic changes of concrete measures of complexity. It turned out that quantitative measures of complexity respond to critical changes in the dynamics of a complex system, which makes them valuable in the diagnostic process and prediction of future changes. Such econophysics approaches give rewarding perspectives for ordinary investors, professional traders, and data analysts who track the state of the investment object (trades) and try to predict a further trend.

Discovered differences between CO_2 emissions futures and the DJIA index, which are also evident in the analyzed complexity measures, in our opinion, are related to the peculiarities of carbon futures. The study of these features encourages us to apply a wider range of methods from the theory of complex systems: fractal, recurrent, quantum, and other measures, which is planned to be done in our subsequent studies.

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