Use of the dynamic mathematical program of GeoGebra in classes in mathematical disciplines in the conditions of blended learning

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Abstract

This article describes one of the universal tools for facilitating blended learning of mathematical disciplines, in particular linear algebra, is the dynamic mathematical program GeoGebra. This program helps to visualize the mathematical objects studied, demonstrate their properties, avoid routine actions, etc. Using GeoGebra makes it possible to improve the quality of the process of solving mathematical problems and improve the professional, mathematical and informational competence of future mathematics teachers. The results of student survey are analyzed and presented regarding the expediency of using this technology in future professional activity. It is established that the, the dynamic mathematical program GeoGebra is a universal tool for organizing blended learning. Using its capabilities, you can both organize the work during the lesson and further independent performance of tasks, and use it at any stage of the lesson.

Keywords

ICT, dynamic mathematical program GeoGebra, future math teachers, blended learning, mathematical competence, informational competence.

1. Introduction

1.1 Formulation of the problem

The rapid development of information technologies, their application in all fields, everyday life, their penetration into the field of education, necessitates computer support of the educational process. That is why the priority of education in the National Doctrine of Education in Ukraine in the XXI century [25] is the introduction of modern information and communication technologies that further improve the educational process, accessibility and effectiveness of education, preparing the young generation for life in the information society. In particular, the document identifies one of the ways to achieve this is by the introduction full-time learning along with distance learning with the use of information and communication technologies along with traditional teaching aids.

The combination of full-time and distance learning tools in the educational process of higher education institutions provides blended learning. Modern students are representatives of the «digital» generation who want to learn quickly, efficiently and dynamically. Blended learning is one way to give them that opportunity.

ICTERI-2021, Proceedings of the 17th International Conference on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer. September 28 – October 2, 2021, Kherson, Ukraine
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1.2 Analysis of recent research and publications

One of the important factors that determine the level of economic and socio-political development of the country is the quality of education of its population, in particular, mathematics. Improving the quality of mathematical knowledge of higher education seekers is one of the most urgent and important tasks modern higher education institutions in general, and pedagogical, in particular, face. The use of blended learning contributes to the improvement of mathematical knowledge of future mathematics teachers, the improvement of the educational process, the rational use of time and resources, and the increase of students' motivation for high-quality mastering of mathematical disciplines.

Some aspects of the use of the model of blended learning in the education system were covered in the works of domestic and foreign scientists: Barna OV [17], Bugaychuk KL [18], Kademiya M.Yu. [21], Korotun OV [22], Krivonos OM [22], Murashchenko TV [24], Nakitina MS [26], Tkachuk GV [27], Moebs S. [10], Weibelzahl S. [10], Charles R. Graham [4], Ismail A. O [5], Mahmood AK [5], Abdelmaboud A. [5], Ghazal S. [3], Al -Samarraie H. [3], Aldowah H. [3], Owston R. [11], York D. [11], Malhotra T. [11].

Today, the term «blended learning» (blended or hybrid learning) in domestic and foreign scientific and educational literature has different interpretations:

- a practical tool for modernization of modern education, that creates new pedagogical methods, which are based on the integration of traditional approaches to the organization of the educational process, where the transfer of knowledge and elearning technologies is happening [22];
- purposeful process of acquiring knowledge, skills and abilities in the context of integration of classroom and extracurricular educational activities of educational subjects based on the use and complementarity of traditional, electronic, distance and mobile learning technologies in the if a student properly manages time, place, routes and pace of study [23];
- learning system, which includes the integration of various forms of learning (full-time learning in classrooms, e-learning and individual learning), which results in a better formation of competencies [26].
- pedagogically balanced combination of traditional, electronic, distance and mobile learning technologies aimed at the integration of classroom and extracurricular learning [24];
- combination of distance and traditional communication in the process of integrated educational activities [10];
- an approach that combines traditional learning with the use of online learning methods (for example, podcasts, materials and activities conducted on the Internet through educational platforms) [4];
- combining traditional formal learning tools: working in classrooms, studying theoretical material - with informal, like, with discussions via e-mail and Internet conferences [20];
- a kind of hybrid method, that combines online learning, traditional and independent learning [28], etc.

The effectiveness and efficiency of the use of blended learning in the process of the students acquiring mathematical knowledge depends on a rational combination of different approaches, methods of presenting, types of work, teaching aids and more. Teaching aids should be modern, reliable, accessible and easy to use. One of the universal tools for facilitating blended learning of mathematical disciplines is the dynamic mathematical program GeoGebra, which helps to visualize the mathematical objects studied, demonstrate their properties, avoid routine actions, etc. Using GeoGebra makes it possible to improve the quality of the process of solving mathematical problems and improve the professional, mathematical and informational competence of future mathematics teachers.

Much attention has been paid to the peculiarities of using the dynamic GeoGebra package in the educational process by foreign scholars, such as: Mailizar [8], Johar R., [8], Jorge Olivares Funes [7], Elvis R. Valero Kari [7], Jorge Andres Olivares Funes [6], Elvis Valero [6], Barahona Avecilla F. [2],

Barrera Cardenas O. [2], Vaca Barahona B. [2], & Hidalgo Ponce B. [2], Arceo-Diaz S. [1], Barrios EEB [1], Maravillas JA [1], Salazar-Torres J. [1], Sugandi AI [14], Bernard M. [14], Majerek D. [9], Septian A. [12] , Suwarman RF [12], E. Monariska [12], R. Sugiarni [12], Septian A. [13], Darhim [13], Prabawanto S. [13], Yismaw A. [15], Wassie G. [15], Awgichew Z., [15], Ziatdinov R. [16], Rakuta V. [16].

Thus, scientists Yismaw A., Wassie G., Awgichew Z. [15] analyzed the possibilities of GeoGebra in process of teaching mathematics and found that the use of this system helps to increase students' interest in learning mathematics. Ziatdinov R., Rakuta V. [16] emphasizes the effectiveness of using the GeoGebra environment for the formation of algebraic thinking skills.

The purpose of the article is to highlight the feasibility and effectiveness of using the dynamic mathematical program GeoGebra in classes in mathematical disciplines (on the example of linear algebra) in a blended learning environment.

2. Methods of the study

Methods used in the research process include: analysis of theoretical sources, generalization of the best pedagogical practices of foreign and domestic specialists of using the dynamic mathematical program GeoGebra in classes in the process of teaching mathematical disciplines (using the example of linear algebra) in a blended learning environment; synthesis, generalization and conceptualization for the development of the main research provisions; interviewing students; generalization of results.

3. Results

One of the most important disciplines of the fundamental cycle of training future teachers of mathematics is linear algebra. The purpose of the discipline «Linear Alge-bra» in pedagogical universities is: to reveal the concept of linear transformation in finite-dimensional spaces, understanding its position and role in the general system of mathematical knowledge and ability to apply it in specific situations, as well as education of algebraic and numerical culture.

When solving a series of problems in linear algebra, it is often time-consuming to perform cumbersome calculations. This leads to a focus on minor details, the performance of ordinary mathematical calculations, and important, significant points are ignored. Therefore, to ensure the automation of solving a certain group of problems, increase the efficiency of learning using the latest techniques and technical teaching aids, in the course of linear algebra it is appropriate and effective to use a dynamic mathematical program GeoGebra.

In addition, the use of the dynamic mathematical program GeoGebra in learning linear algebra is reasonable due to many other factors, including:

- intensification of the educational process, in connection with which more and more hours for the study of linear algebra are allocated for individual learning. So, for example, in Pavlo Tychyna Uman State Pedagogical University, the linear algebra curriculum dedicated 180 hours to the discipline, out of them 94 hours are set aside for independent study of educational material by students.
- the difficulty of studying linear algebra by students, in particular due to the need to cover a lot of theoretical material, therefore there is a need to constantly intensify the educational activities of students and promote interest in the discipline.
- the need to visualize educational material, to quickly and effectively evaluate the results of educational activities, etc.

GeoGebra software can be used to increase the efficiency of individual learning of linear algebra, and during lectures and practical classes as part of full-time and distance and blended learning. During practical training in linear algebra, GeoGebra can act as an intermediary between the teacher and students, acting as a simulator in solving certain types of problems. For example, the applet «Gauss-Jordan – Latest» can be used during the practical lesson for first-year students majoring in 014.04 Secondary Education (Mathematics) on «Systems of linear equations», aiming to develop practical skills and ability to solve systems of linear equations by Gauss and Jordan-Gauss (Fig. 1) [19].

Figure. 1. The GeoGebra applet is designed to solve systems of linear equations

In a practical lesson, before students use the applet on their own, the teacher needs to show all its possibilities by a concrete example. One must first propose to solve a system of linear equations that has one solution. By identifying a single solution using an applet, students can check the notebook on their own. For example, one can consider the solution of the system $\begin{cases} 2x_1 + 6x_2 + 7x_3 = 25, \\ x_1 + 2x_2 + 3x_3 = 10, \\ x_1 + 4x_2 + 6x_3 = 17. \end{cases}$ The functionality of the simulator involves the introduction of any system (Fig. 2).

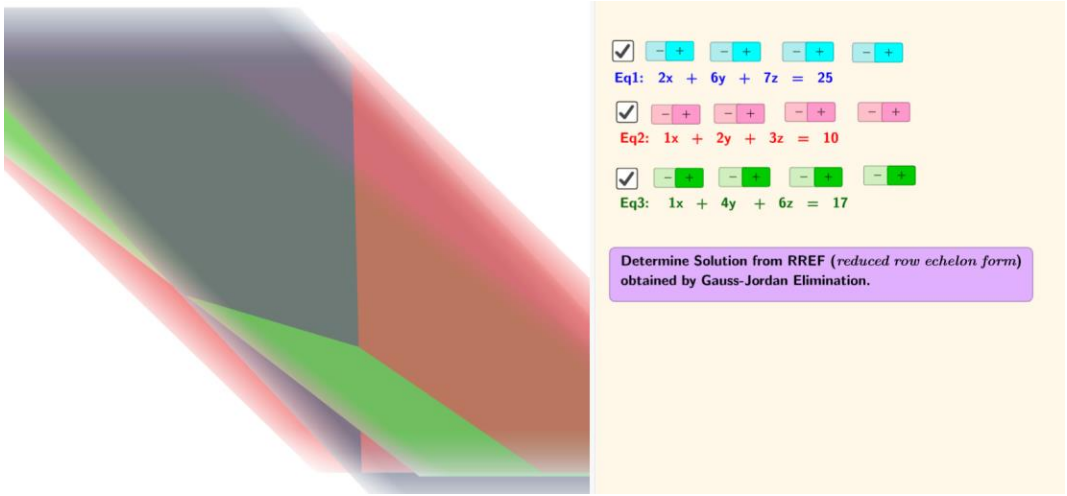
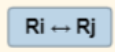


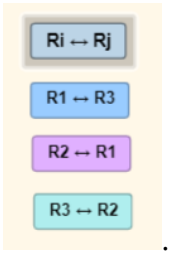
Figure 2. Writing a system of linear equations

Using, a specific example, the teacher shows that with this applet you can work the following elementary transformations on the rows of the matrix:

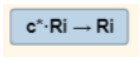
1. Permutation of two lines in places.

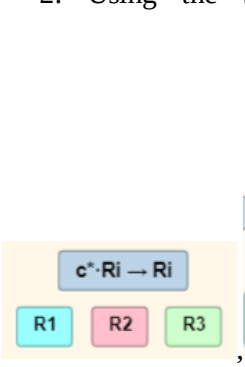
Rearrangement of two lines in places can be done using the appropriate button 

, which

after pressing takes the following form: 

. At this stage, students should be given useful advice that it is desirable to choose the leading element of the first line so that it is not zero and is equal to ± 1. Rearranging lines in places will help to do this.

2. Using the button 

(which after pressing is transformed into a button 

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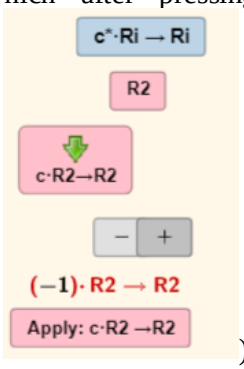
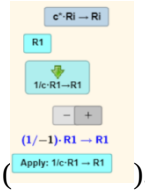
, 

) you can multiply the selected line by any number. At the stage of multiplying a line by a number, the teacher draws attention to the fact that there may be cases when it is possible to reduce both parts of the equation by a number (say c), which will simplify the following calculations. This can be done by multiplying the row of the extended system matrix by the number $\frac{1}{c}$ (

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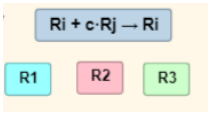
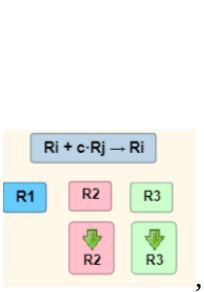
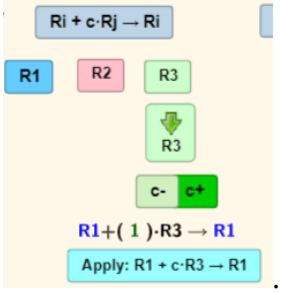
3. Add a line multiplied by some number to another line.

You can add another number multiplied by a number to one line using the button 



, 

. At this stage, the teacher needs to explain why the leading element of the first line was made the number ± 1, demonstrating with the help of the applet of addition to each line of the first, multiplied by the corresponding coefficients.

By reducing the matrix to a stepped shape, the teacher can use this simulator to continue to transform the matrix of the system into a consolidated stepped shape, reminding students of the difference in solving systems of linear equations by the Gaussian and Jordan-Gaussian methods.

After writing down the solutions of this system, students can be asked to solve two more systems

of linear equations, for example, the system $\begin{cases} x_1 + 2x_2 + 3x_3 = 4, \\ 2x_1 + x_2 - x_3 = 3, \\ 3x_1 + 3x_2 + 2x_3 = 7. \end{cases}$ $\begin{cases} x_1 + 2x_2 + 3x_3 = 4, \\ 2x_1 + 4x_2 + 6x_3 = 3, \\ 3x_1 + x_2 - x_3 = 1. \end{cases}$ one of which has many solutions, and the other has no solutions.

If the lesson is in remote or mixed modes, then the demonstration of the use of the dynamic mathematical program GeoGebra should be done screen sharing using video conferencing programs, such as Google Meet, Zoom, etc.

Despite the simple and intuitive tools of the applet, before letting the students to use it independently using it independently, the teacher must not only introduce students to the functionality by solving specific problems, but also provide students with instruction cards. Instruction cards should be compiled in such a way that the student can work independently with the service without the help of the teacher (see Fig. 3). To do this, one needs to include QR-codes with a link to the applet and other useful links, as well as a specific example, to show the stages of solving a system of linear equations in the instruction cards. Since some students may have difficulty understanding the purpose of certain buttons, it is desirable that the instruction cards have a translation of the inscriptions.

We offer to provide students with a bank of tasks (3-6 tasks), which they must perform with the help of a simulator. This type of work will contribute to the formation of practical skills and abilities to reduce systems to row echelon form and reduced row echelon form, to solve systems of linear equations by the Jordan-Gauss method. This type of work with this simulator will teach students to perform methodically correct transformations, without wasting time on simple calculations. However, as practice shows, students make the most mistakes performing simple calculations, which leads to incorrect solutions. Therefore, after working in the program GeoGebra, it is necessary to offer students to perform several more tasks in notebooks, in order to prevent errors in actions with coefficients.

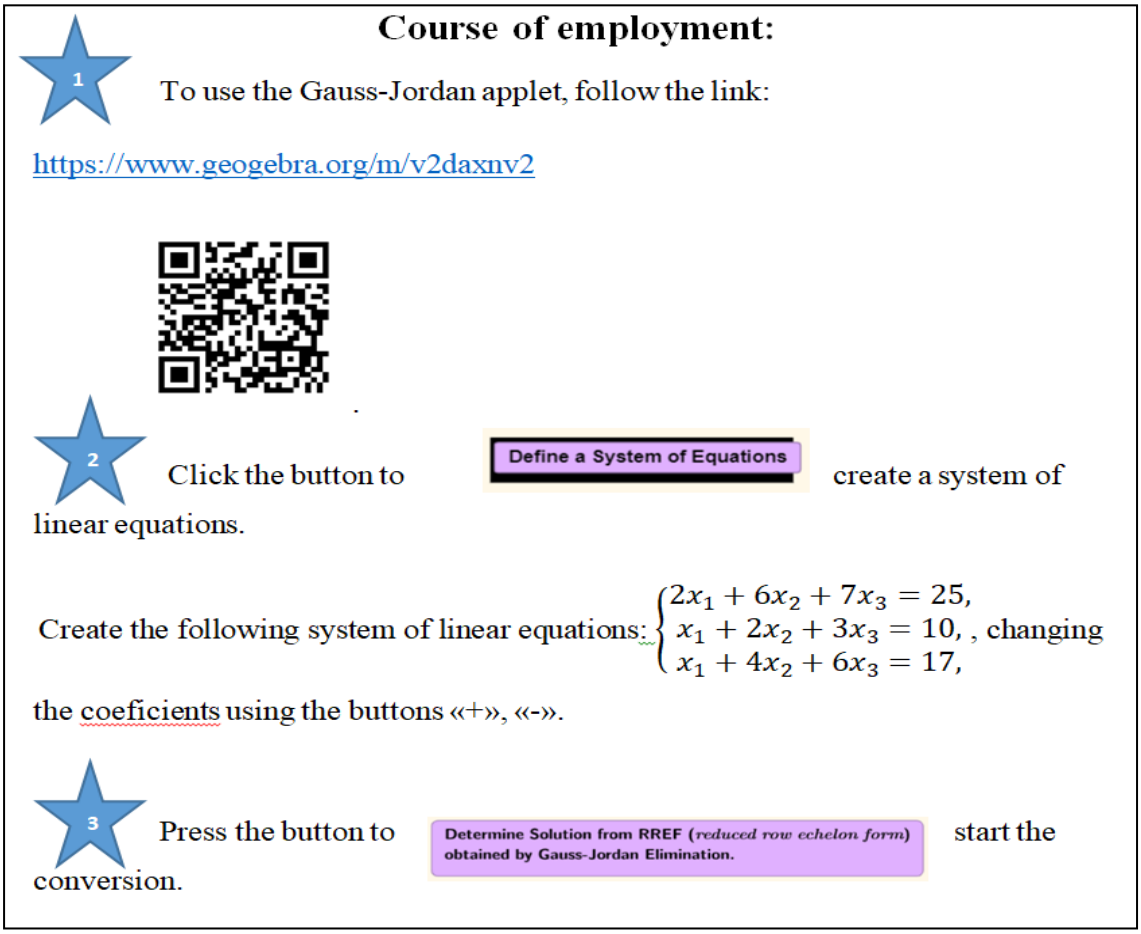


Figure. 3. Fragment of the instruction card

Availability of GeoGebra program promotes realization of an educational path of applicants of higher education. Students can work with the program not only during classes according to the schedule, but also in extracurricular, convenient time. They can choose the appropriate composition and number of tasks, the resolution of which, in their opinion, is necessary and sufficient (depending on individual capabilities and needs) for the comprehensive assimilation of educational material and the development of appropriate computational ability and skills.

There are many advantages to using the «Gauss-Jordan - Latest» applet [19] during practical problems in linear algebra. The main ones are:

- increase of the efficiency of formation of practical skills and abilities of students to solve systems of linear equations by Gauss and Jordan-Gauss methods by developing a clear, methodically correct algorithm of students' actions;
- significant time savings due to elimination of routine calculations;
- involving students' cognitive activity;
- work with the simulator can be done not only with the help of desktop computers or laptops, but also with the help of tablets or smartphones, which allows for teaching in the places other than computer classrooms. Connected to the Internet, students can have online can complete tasks as effectively as during face to face classes. Thus, the GeoGebra software becomes an indispensable assistant in blended learning;
- GeoGebra applets are easy to integrate into electronic manuals or textbooks.

However, there are significant disadvantages of using this simulator, among them:

- the need for tools to work with the program GeoGebra, as well as connection to the Internet;
- insufficient level of computer literacy and methodical preparation for practical work using the dynamic mathematical program GeoGebra;
- students use programs for other purposes, which can lead to poor learning of educational material.

After conducting classes in linear algebra in a blended learning environment using the dynamic mathematics program GeoGebra, students were surveyed about the interest and effectiveness of using this program. Students were presented with a QR code, with the link to the questionnaire the students could answer using their phones and tablets.

The survey can be conducted both synchronously (in the audience, «here and now») and asynchronously - at any time within a specified interval of the survey.

- 9 students of the Faculty of Physics, Mathematics and Informatics of Pavlo Tychyna Uman State Pedagogical University took part in the survey. The results of the survey are presented in Figures 4 and 5.

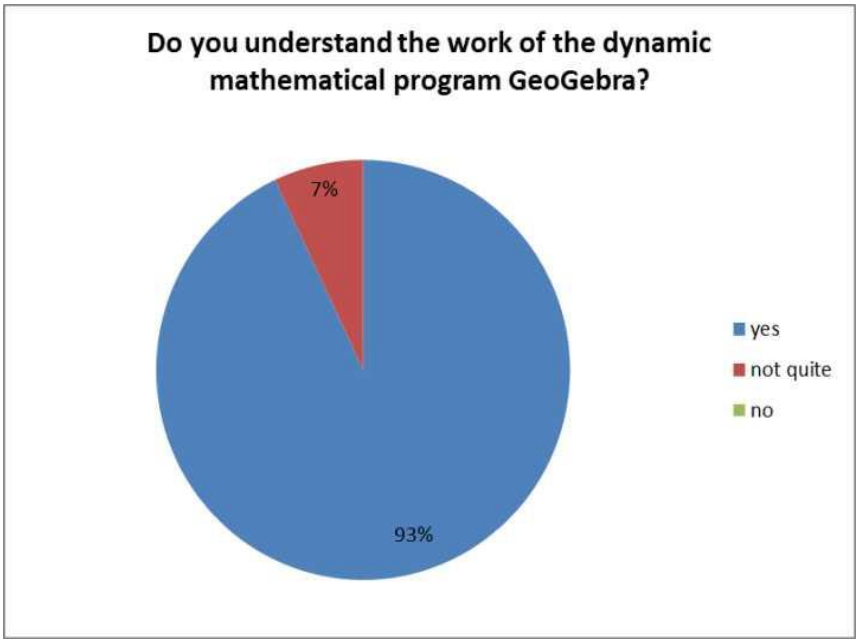


Figure 4. Results of students' questionnaire

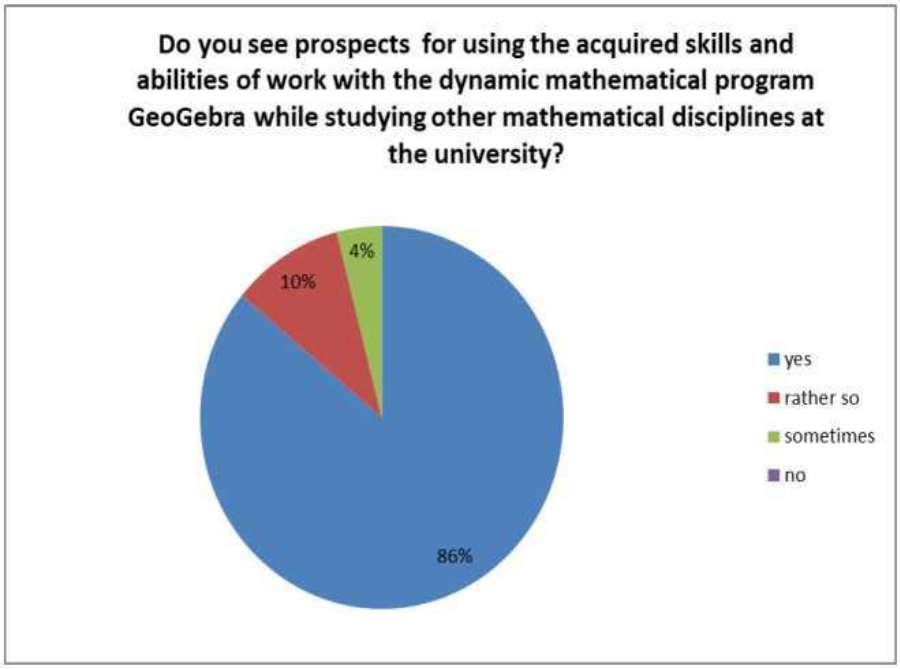


Figure 5. Results of students' questionnaire

The interaction of participants in the educational process with various computer based teaching aids, so-called teaching aids, should be used in classes in professional disciplines as often as possible, so that future teachers can successfully and skillfully implement them in their professional activities. After all, such computer-based learning tools allow to optimize and bring excitement and interest in the educational process, and motivate students.

The use of GeoGebra should not be limited to lectures or practical classes, one can ask students to do homework on their own in a notebook, and only then check in the program. One can use the proposed program to effectively check the completed task, when the teacher is the one checking, or when students check each other's work in pairs or groups. Thus, the dynamic mathematical program GeoGebra is a universal tool for organizing blended learning. Using its capabilities, one can both organize the work during the class at first, and then implement it into independent learning, and use it at any stage of the lesson.

4. Conclusions

Dynamic mathematical program GeoGebra, that clearly demonstrates the formal, algorithmic nature of problem allows students to form an algorithmic style of thinking, master modern information and communication technologies and get a powerful tool for solving problems. One of the effective means of increasing the effectiveness of teaching mathematical disciplines, including linear algebra, is the pedagogically balanced use of computer-based learning systems. The use of the dynamic mathematical program GeoGebra during the blended learning of mathematical disciplines brings the educational process to a qualitatively new level, provides an opportunity to build the educational trajectory of each student individually, taking into account his/her abilities and needs.

Further research should focus on the use of dynamic mathematical program Geo- Gebra in teaching students of other disciplines.

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